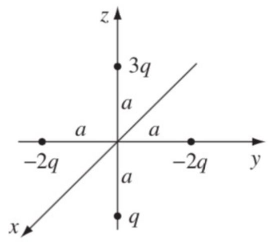
PHY209 Electromagnetism
Assignment 5

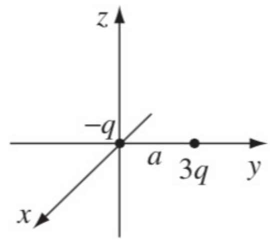
Handed out: September 18, 2019

Problem 1



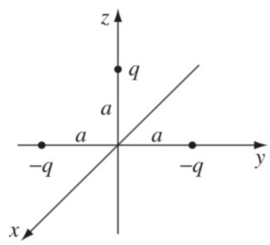
Four particles (one of charge q, one of charge $3q$, and two of charge $-2q$) are placed as shown in Figure, each a distance a from the origin. Find a simple approximate formula for the potential, valid at points far from the origin. (Express your answer in spherical coordinates.)

Problem 2



Two point charges, $3q$ and $-q$, are separated by a distance a. For the arrangement in Figure, find (i) the monopole moment, (ii) the dipole moment, and (iii) the approximate potential (in spherical coordinates) at large r (include both the monopole and dipole contributions).

Problem 3



Three point charges are located as shown in Figure, each a distance a from the origin. Find the approximate electric field at points far from the origin. Express your answer in spherical coordinates, and include both the monopole and dipole contributions.

Problem 4

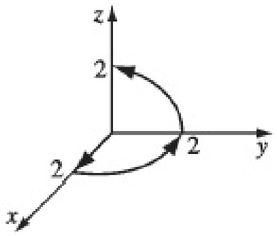
Show that the electric field of a (perfect) dipole can be written in the coordinate-free form

$$\boldsymbol{E}_{dip}(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0 r^3}[3(\boldsymbol{p}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} - \boldsymbol{p}] \tag{1}$$

Problem 5

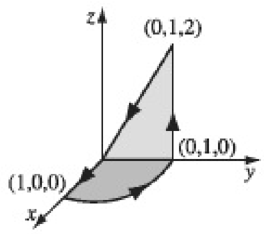
Express the unit vectors $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\theta}}$, $\hat{\boldsymbol{\phi}}$ in terms of $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$, $\hat{\boldsymbol{z}}$. Check your answers several ways ($\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{r}} = 1$, $\hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = 0$, $\hat{\boldsymbol{r}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$, ...). Also work out the inverse formulas, giving $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$, $\hat{\boldsymbol{z}}$ in terms of $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\theta}}$, $\hat{\boldsymbol{\phi}}$ (and θ, ϕ).

Problem 6



Compute the gradient of the function $T = r(\cos\theta + \sin\theta\cos\phi)$. Test the gradient theorem for this function, using the path shown in Figure, from $(0,0,0)$ to $(0,0,2)$.

Problem 7



Compute the line integral of

$$\boldsymbol{v} = (r\cos^2\theta)\hat{\boldsymbol{r}} - (r\cos\theta\sin\theta)\hat{\boldsymbol{\theta}} + 3r\hat{\boldsymbol{\phi}} \tag{2}$$

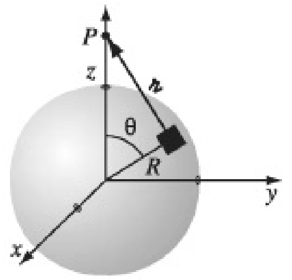
around the path shown in Figure (the points are labeled by their Cartesian coordinates).

Problem 8

The integral $\boldsymbol{a} = \int_S d\boldsymbol{a}$ is sometimes called the vector area of the surface S. If S happens to be flat, then $|\boldsymbol{a}|$ is the ordinary (scalar) area. Find the vector area of a hemispherical bowl of radius R.

Problem 9

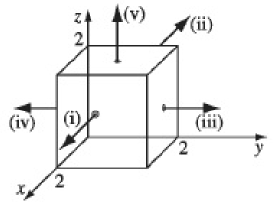
Find the potential and electric field a distance z from the center of a spherical surface of radius R (Figure) that carries a uniform charge density σ. Treat the case $z < R$ (inside) as well as $z > R$ (outside). Express your answers in terms of the total charge q on the sphere.



Problem 10

Use your result in Prob. 9 to find the potential and the field inside and outside a solid sphere of radius R that carries a uniform volume charge density ρ. Express your answers in terms of the total charge of the sphere, q. Draw a graph of $|\boldsymbol{E}|$ as a function of the distance from the center. Compute the gradient of V in each region, and check that it yields the correct field. Sketch $V(r)$.

Problem 11

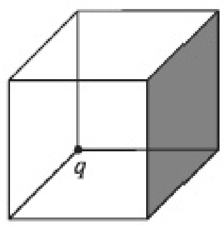


Calculate the surface integral of $\boldsymbol{v} = 2xz\hat{\boldsymbol{x}} + (x+2)\hat{\boldsymbol{y}} + y(z^2-3)\hat{\boldsymbol{z}}$ over five sides (excluding the bottom) of the cubical box (side 2) in Figure. Let 'upward and outward' be the positive direction, as indicated by the arrows.

Problem 12

Suppose the electric field in some region is found to be $\boldsymbol{E} = kr^3\boldsymbol{r}$, in spherical coordinates (k is some constant). Find the total charge contained in a sphere of radius R, centered at the origin.

Problem 13



A charge q sits at the back comer of a cube, as shown in Figure. What is the flux of $\boldsymbol{E}$ through the shaded side?